A CAD Program and Equations for System Phase and Amplitude Noise Analysis

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ABSTRACT

A novel method for calculating the amplitude and phase noise of component chains is described. The equations for oscillator, amplifier, multiplier, and bandpass filters are discussed. These equations include the upconversion of 1/f noise. A user friendly CAD program is described. This work is important for the calculation of amplifier chain noise, fiber optic receiver jitter, and general system noise analysis.

INTRODUCTION

A knowledge of the phase and amplitude noise produced in a chain of components is important for the calculation of amplifier chain noise, fiber optic receiver jitter, and general system noise analysis. The simplest method of rigorously accounting for the amplitude and phase noise of a chain of components is to construct a modulation transfer matrix for each component and then multiply these matrices just as you would multiply the ABCD matrices of circuit components [1]. At the beginning of each chain is an oscillator (source). The oscillator amplitude and phase noise is calculated from closed form expressions derived in reference [2]. These equations are repeated in the following section and include such affects as device saturation characteristics, 1/f noise upconversion, and amplifier AGC affects.

This paper introduces a new derivation of the modulation noise which allows low frequency (1/f and power supply) modulations to be included. This was previously thought to be incompatible with matrix based modulation analysis [1]. This paper also describes how this analysis can be performed using a low cost, commercially available simulation package [3]. Computers, languages, and utilities have improved a great deal in the past few years. Software such as the *Extend* package by *Imagine That!*, for Macintosh™ computers, allows the engineer to focus on writing the equations for the analysis since the software takes care of creating user friendly dialog boxes, schematic capture (see Fig. 1), plots, and data importing/exporting. This sort of simulation package finally makes possible the efficient generation of analysis software by engineers. For example, the library of components used for this modulation noise analysis was constructed in approximately one man-week.

THEORY

A matrix based modulation noise analysis technique allows simple derivations of the effects of nonlinear components on noise. The basic assumptions are that the noise has a gaussian distribution and is stationary. While stationarity has never been proven or disproven for the flicker (1/f) noise we deal with in practice, the assumption appears to work in all cases of interest.

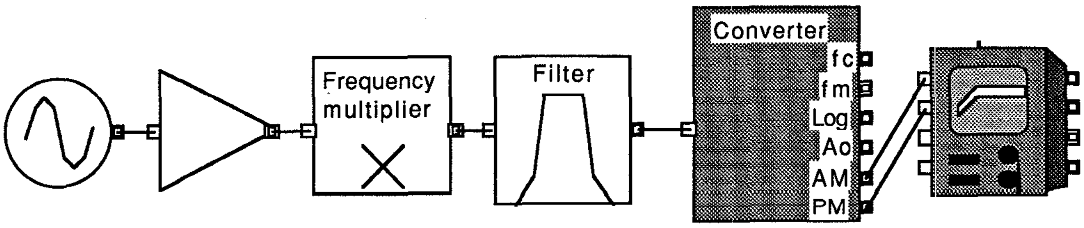


Fig 1. Screen display of oscillator-multiplier chain example.

$$\begin{bmatrix} m_{out} \\ \beta_{out} \end{bmatrix} = \begin{bmatrix} T_{aa} & T_{ap} \\ T_{pa} & T_{pp} \end{bmatrix} \begin{bmatrix} m_{in} \\ \beta_{in} \end{bmatrix} + \begin{bmatrix} m_{add} \\ \beta_{add} \end{bmatrix} \qquad (1)$$

where **m** and β are the amplitude and phase modulations, respectively. The subscripts **in** and **out** indicate input and output modulation, while the subscript **add** indicates modulation added in by a noisy component. The **Ts** represent modulation transfer coefficients. These coefficients are the amplitude to amplitude (AM-AM), PM-AM, AM-PM, and PM-PM modulation conversion coefficients for the device at a given modulation frequency (see Eqn 1). These matrices are easily constructed by considering the modulation noise as a small perturbation of the signal (carrier), and following the methods described in several books and articles [3-5]. Key simplifying aspects of analyzing noise in nonlinear devices are considering the noise in the frequency domain, using many very small bandwidths of noise (so each may be approximated by an equivalent sinusoid), and assuming there exists a carrier signal which is much larger than the noise signals [3,4,6].

$$V_o(t) = V_c \{1 + m(t)\} \cos[\omega_c t + \beta(t)] \qquad (2)$$

where $\mathbf{m}(t) = \mathbf{m} \cos(\omega_m t)$, $\beta(t) = \beta \cos(\omega_m t)$ and $\omega_\mathbf{m}$ is the modulation frequency. **m** and β are implicit functions of $\omega_\mathbf{m}$.

The derivation begins with an amplitude and phase modulated signal as given in Eqn. 2. This equation may be expanded to a carrier and two sets of sidebands if the modulations are small (Eqn 3). One set of sidebands is in phase and gives the amplitude modulation, while the other is out of phase and gives the phase modulation. The modulation transfer matrix elements are derived by analyzing the output of a nonlinear device for either pure AM or PM input [4]. Any asymmetry in the output sidebands indicates the presence of both modulations.

$$\begin{aligned} V_o(t) &= V_c \cos(\omega_c t) \\ &\quad + V_c \frac{m}{2} [\cos(\omega_c t + \omega_m t) + \cos(\omega_c t - \omega_m t)] \\ &\quad + V_c \frac{\beta}{2} [\cos(\omega_c t + \omega_m t) - \cos(\omega_c t - \omega_m t)] \qquad (3) \end{aligned}$$

A simple example of this would be the transfer matrix for an ideal frequency multiplier. The input modulation would be as in Eqn. 2, but the output modulation would be given by Eqn. 4. Here it is assumed the multiplier has no loss or saturation effects, but only a frequency multiplication of **n**. By expanding Eqn. 4 and comparing it to an expanded version of Eqn. 2 (see Eqn. 3) we can determine the modulation transfer matrix. The transfer matrix resulting from this is shown in Eqn. 5.

$$V_o(t) = V_c \{1 + m(t)\} \cos[\, n\{\omega_c t + \beta(t)\}] \qquad (4)$$

$$\mathbf{T}(\omega_m) = \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix} \qquad (5)$$

The analysis of amplifier noise contributions is a bit more complicated but is very important for phase noise specifications. The modulation transfer matrix of an ideal amplifier is the identity matrix. The added modulation comes from the noise vector in Eqn. 1. This noise modulation vector is made up of high frequency device noise (from noise figure measurements), and low frequency modulation effects. The high frequency amplitude and phase modulation may be calculated as $\sqrt{\{4kTBR(F-1)\}}/V_c$ where **k** is Boltzmann's constant, **T** is temperature in degrees Kelvin, **B** is bandwidth, **R** is the system resistance (50Ω), **F** is the device noise figure, and $\mathbf{V_c}$ is the rms carrier voltage [2]. The low frequency noise (voltage or current) is upconverted to high frequencies through device nonlinearities. This upconversion will have a real part (amplitude) and an imaginary part (phase). The upconversion may be calculated as a carrier dependent conversion coefficient (typically proportional to the carrier for carriers below device saturation), or as the change in the device gain or phase with bias as shown in Eqn. 6. The total amplifier noise matrix is the rms sum of these two noise contributions.

$$m = v_n \frac{G_{up}}{V_c} \text{ or } v_n \frac{\partial Gain}{\partial v_n}, \text{ and } \beta = v_n \frac{\theta_{up}}{V_c} \text{ or } v_n \frac{\partial \theta}{\partial v_n} \qquad (6)$$

$$\begin{bmatrix} m_{add} \\ \beta_{add} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{4kTR(F-1)}}{V_c} + v_n \frac{\partial A}{\partial v_n} \\ \frac{\sqrt{4kTR(F-1)}}{V_c} - v_n \frac{\partial \theta}{\partial v_n} \end{bmatrix} \qquad (7)$$

where $\mathbf{G_{up}}$ is the upconversion, $\theta_{\mathbf{up}}$ is the phase upconversion, θ is the component phase, **A** is the component gain, and $\mathbf{v_n}$ is the low frequency noise. It must be remembered that $\mathbf{v_n}$ is a random variable and can only be added to the high frequency noise in a mean square sense.

Closed form expressions for oscillator AM and PM noise are given below as derived in [2]. In equations (8) and (9) s stands for the gain compression (slope of the gain curve, ideally *-1*), σ stands for the left half plane pole location of the oscillator (typically 10^{-9} or less), and τ_g stands for the group delay (approximately $2Q/\omega_0$) of the oscillator. Both the gain compression factor and the group delay may be functions of frequency, depending on if the oscillator has AGC and how complicated the loop transfer function is, respectively.

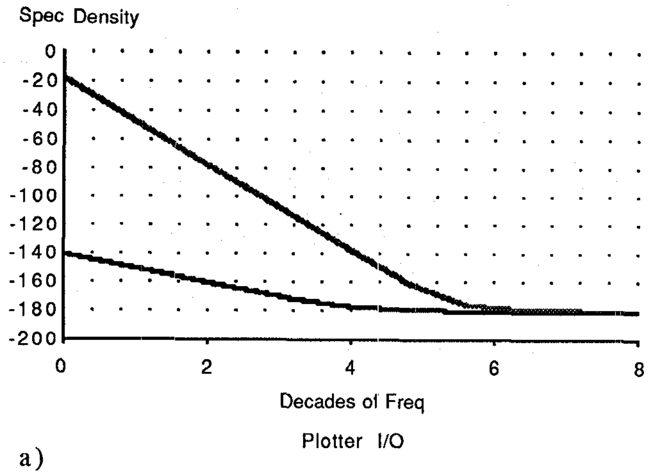
$$S_A(\omega_m) = \frac{1}{s^2 + (\omega_m \tau_g)^2}\left[\frac{2kTRF}{V_c^2} + v_n^2\left(\frac{\partial A}{\partial v_n}\right)^2\right] \quad (8)$$

$$S_\theta(\omega_m) = \frac{1}{\sigma^2 + (\omega_m \tau_g)^2}\left[\frac{2kTRF}{V_c^2} + v_n^2\left(\frac{\partial \theta}{\partial v_n}\right)^2\right] \quad (9)$$

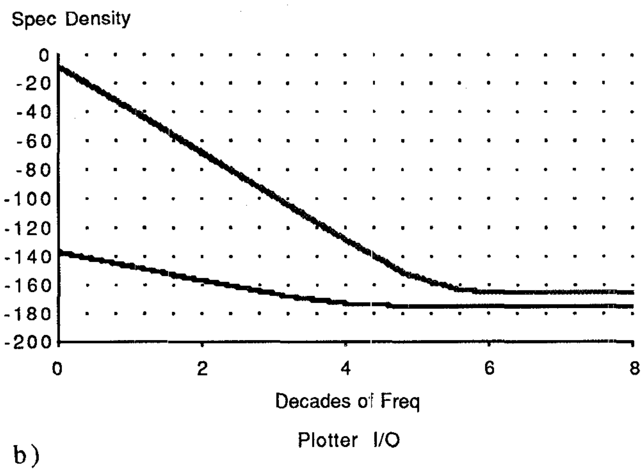
Other components include a bandpass filter and a jitter calculator. The bandpass filter derivation is too long to include here but is described in [1,2]. The jitter calculation is obtained by integrating the FM noise ($\omega^2 S_\theta$) up to the system bandwidth [7].

APPLICATIONS

The CAD program was created by programming the above equations into blocks (or components) within *Extend*. Each component simply takes an array of numbers from its input connector, does calculations in a C like language, and sends a new array of modulation and amplitude information out the output connector (see Fig. 1). The blocks are chosen from a menu, dragged into place, and connected in a "point and click" style. If a user clicks twice on any block they are presented with a dialog box of parameters and help information. The program performs one cycle through the chain of blocks for each modulation frequency (set by the oscillator block) and the result is plotted on the plotter block. The converter block merely converts the array of input data to many single valued outputs which *Extend's* generic plotter can then use.



a)



b)

Fig. 2. Noise plots (dbc/Hz versus frequency). a) Plot of 10GHz example oscillator phase (upper line) and amplitude noise. b) Plot of phase and amplitude noise after passing through the multiplier chain of Fig. 1. Note almost 10db increase in phase noise due to the multiplication, and the increase in AM noise from the amplifier. The 6th decade of frequency is 10^6 Hz or 1MHz.

The above diagram shows a typical plot of amplitude and phase noise from an oscillator (Fig. 2a). Figure 2b shows the oscillator noise after it has been passed through the chain of Figure 1 (the multiplier ratio is 3). Many types of analysis can be done with this program. For example it can be easily shown that when similar amplifiers are cascaded the phase noise of each amplifier contributes equally to the total chain phase noise provided none of the amplifiers are limiting. However, if the last amplifiers in the chain use larger FETs, or have FETs optimized for linearity, their phase noise contribution will be less because of less flicker noise or less nonlinearity respectively [2].

CONCLUSIONS

The method of analyzing system amplitude and phase noise presented in this paper is both simple and rigorous. This method extends earlier works to include 1/f noise effects. The incorporation of these tedious calculations into a user friendly CAD program is a major step toward reducing the engineering time involved in this sort of analysis.

ACKNOWLEDGEMENT

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REFERENCES

[1] A. Takaoka, and K. Ura, "Noise Analysis of Nonlinear Feedback Oscillator with AM-PM Conversion Coefficient," IEEE Trans MTT, pp. 654 - 662, Jun 1980.

[2] A.N. Riddle, *Oscillator Noise: Theory and Characterization*, P.h.D. Dissertation, North Carolina State University, 1986.

[3] Imagine That!, (Macintosh), 7109 via Carmela, San Jose, CA, 95139, (408) 365-0305

[4] W.F. Egan, "The Effects of Small Contaminating Signals in Nonlinear Elements Used in Frequency Synthesis and Conversion," Proc. IEEE, pp. 797 - 811, July 1981.

[5] A. Papoulis, *The Fourier Integral and Its Applications*, McGraw-Hill, New York, 1963.

[6] A.B. Carlson, *Communication Systems*, McGraw-Hill, New York, Jul 1975.

[7] W.P. Robins, *Phase Noise in Signal Sources*, Peter Peregrinus Ltd, Exeter, 1982.